Exercise 44

For the following exercises, solve the equations over the complex numbers.

$$x^2 - 2x + 4 = 0$$

Solution

The two terms with x, x^2 and $2x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -2$, which means $B = -1$ and $B^2 = 1$. Add and subtract 1 on the left side and apply the identity.

$$(x^2 - 2x + 1) + 4 - 1 = 0$$

$$(x + (-1))^2 + 3 = 0$$

$$(x - 1)^2 + 3 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 3 from both sides.

$$(x - 1)^2 = -3$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x-1)^2} &= \sqrt{-3} \\ &= \sqrt{(-1)3} \\ &= \sqrt{-1}\sqrt{3} \\ &= i\sqrt{3}\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 1$.

$$|x - 1| = i\sqrt{3}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x - 1 = \pm i\sqrt{3}$$

Add 1 to both sides.

$$x = 1 \pm i\sqrt{3}$$

Therefore,

$$x = \{1 - i\sqrt{3}, 1 + i\sqrt{3}\}.$$